Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

http://sam.nitk.ac.in/ sam@nitk.edu.in

MA110 - Engineering Mathematics-1
Problem Sheet - 8

Double Integrals in Cartesian Coordinates

---

1. Approximate the integral

   (a) $\iint_R (4x^3 + 6xy^2)dA$ over the rectangle $R = [1,3] \times [2,1]$ by partitioning R into six unit squares $R_1, R_2, \ldots, R_6$ and by selecting each (x_i, y_j) as the lower left corner of the rectangle R_j.

   (b) $\iint_R (4x^2 + y^2)dA$ over the rectangle $R = [0,2] \times [0,3]$ by partitioning R into six unit squares $R_1, R_2, \ldots, R_6$ and by selecting each (x_i, y_j) as the upper right corner of the rectangle R_j.

2. Evaluate $\iint_R \sin(y^3)dA$, where R is the region bounded by $y = \sqrt{x},\ y = 2,$ and $x = 0$.

3. What region R in the xy-plane maximizes the value of

$$\iint_R (4 - x^2 - 2y^2)\, dA \ ?$$

   Give reasons for your answer.

4. Evaluate the follwing improper integrals as iterated integrals:

   (a) $\iint_D \frac{1}{(x+y)^2}\, dA,$ where D is the region $0 \le x \le 1,\ \ 0 \le y \le x^2$.

   (b) $\iint_R e^{-x^2}\, dA$, where R is the region where $x \ge 0$, and $-x \le y \le x$.

5. Find the volume of the solid enclosed by the cylinders $z = x^2,\ \ y = x^2$ and the planes $z = 0,\ \ y = 4$.

6. Find the average distance from points in the quarter-disk $x^2 + y^2 \le a^2,\ x \ge 0,\ y \ge 0$, to the line $x + y = 0$.

7. Evaluate following iterated integrals:

   (a) $\int_1^3 \int_{-y}^{2y} xe^{y^3} dxdy$ (b) $\int_0^2 \int_{-x}^{x} e^{-x^2} dydx$ (c) $\int_1^2 \int_0^{x^2} \frac{y^2}{x} dydx$

8. Using double integral, find the volume of the solid bounded by $x^2 + y^2 = 16$ and $y^2 + z^2 = 16$.

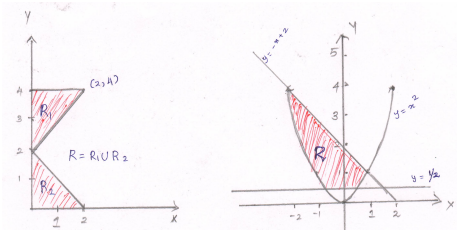
9. Find the volume of the solid bounded by the parabolic cylinder $x^2 = 4y$ and the planes $z = 0$ and $5y + 9z - 45 = 0$.

10. Find the volume of the solid in the first octant bounded by the circular paraboloid $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 4$ and the co-ordinate planes.

11. Change the order of integration in the following integrals (sketch the regions):

(a) $\int_0^a \int_0^{\sqrt{2ay-y^2}} f(x,y)dxdy$ (b) $\int_0^1 \int_{y^2}^{y^{1/3}} f(x,y)dxdy$ (c) $\int_0^1 \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dxdy$

12. Evaluate $\iint_R xy^2 dA$, where R is the region as shown below:



13. Evaluate (using Cartesian coordinates) $\iint_R (x^2 + x^4 y)dA$ where $R = \{(x,y)/1 \leq x^2 + y^2 \leq 4\}$.

*****